Compensating Variation, Equivalent Variation, and Consumer Surplus

Suppose a consumer has a utility function $u(x, y) = x^{1/2}y^{1/2}$. Let p_x and p_y be the prices of goods x and y, respectively, and let I be the individual's income.

1. Obtain the demand functions (Marshallian demands).
2. Obtain the indirect utility function.
3. Obtain the utility level of the individual when both prices are \$1 (as a function of I).
4. Obtain the utility level of the individual when the price of x rises to \$2 (as a function of I).
5. Using your answers to the previous points, find the compensating variation (CV) of the increase in the price of x from \$1 to \$2 (as a function of I).
6. The equivalent variation (EV) of the increase in the price of x is the change in the individual's income (if the price of x does not increase) that produces the same utility loss as the price increase. Using your answers to points (d) and (e), again, find the value of the EV of the increase in the price of x from \$1 to \$2 (as a function of I).
7. Suppose $I = 100$ and calculate the value of both variations. Which is greater?
8. Prove that the change in consumer surplus falls between these two values.

Solutions

1. To find the Marshallian demand functions, we set up the Lagrangian:

$$\mathcal{L} = x^{1/2}y^{1/2} + \lambda(I - p_x x - p_y y)$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{2}x^{-1/2}y^{1/2} - \lambda p_x = 0$$
$$\frac{\partial \mathcal{L}}{\partial y} = \frac{1}{2}x^{1/2}y^{-1/2} - \lambda p_y = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - p_x x - p_y y = 0$$

From the first two conditions, we get:

$$\frac{1}{2}x^{-1/2}y^{1/2} = \lambda p_x \quad \Rightarrow \quad \lambda = \frac{1}{2p_x}x^{-1/2}y^{1/2}$$
$$\frac{1}{2}x^{1/2}y^{-1/2} = \lambda p_y \quad \Rightarrow \quad \lambda = \frac{1}{2p_y}x^{1/2}y^{-1/2}$$

Setting the expressions for λ equal, we have:

$$\frac{1}{2p_x}x^{-1/2}y^{1/2} = \frac{1}{2p_y}x^{1/2}y^{-1/2}$$
$$\frac{y}{x} = \frac{p_x}{p_y} \quad \Rightarrow \quad y = \frac{p_x}{p_y}x$$

Substitute $y = \frac{p_x}{p_y}x$ into the budget constraint:

$$I = p_x x + p_y \left(\frac{p_x}{p_y}x\right)$$
$$I = p_x x + p_x x$$
$$I = 2p_x x$$
$$x = \frac{I}{2p_x}$$

Similarly, for y:

$$y = \frac{p_x}{p_y}x = \frac{p_x}{p_y}\frac{I}{2p_x}$$
$$y = \frac{I}{2p_y}$$

2.

$$V = \sqrt{\frac{I}{2p_y}}\sqrt{\frac{I}{2p_y}} = \frac{I}{2\sqrt{p_x p_y}}$$

3.

$$U_0 = \frac{I}{2}$$

4.

$$U_1 = \frac{I}{2\sqrt{2}}$$

5. The CV must be such that after receiving it, the individual remains at the same utility level as before the price increase of x, U_0. That is, it must satisfy

$$\frac{I}{2} = \frac{I + CV}{2\sqrt{2}}$$

Therefore,

$$CV = (\sqrt{2} - 1)I$$

6.

$$\frac{I - EV}{2} = \frac{I}{2\sqrt{2}}$$

$$EV = \frac{(\sqrt{2} - 1)I}{\sqrt{2}}$$

7.

$$CV = (\sqrt{2} - 1)100 = 41.42$$

$$EV = \frac{(\sqrt{2} - 1)100}{\sqrt{2}} = 29.29$$

Compensating Variation (CV) measures the amount of money needed to compensate the consumer after the price change to bring them back to their initial utility level. Equivalent Variation (EV) measures the amount of money needed to be taken away from the consumer before the price change to bring them to the lower utility level they will reach after the price increase. When CV is greater than EV, it indicates that the consumer needs more compensation after the price increase to reach the initial utility level than what would be equivalent to the utility decrease before the price change. This implies a significant substitution effect, suggesting that the consumer finds it relatively difficult to substitute away from the good whose price has increased.

8.

$$\Delta CS = \int_1^2 \frac{I}{2p_x} dp_x = \int_1^2 \frac{50}{p_x} dp_x = 50\ln(2) - 50\ln(1) = 34.65$$